// University of Texas Bulletin

No. 2642: November 8, 1926

The Texas Mathematics Teachers' Bulletin

Volume XI, No. 1

Edited by

C. D. RICE
Associate Professor of Applied Mathematics

PUBLISHED BY
THE UNIVERSITY OF TEXAS
AUSTIN

Publications of the University of Texas

Publications Committees:

The University publishes bulletins four times a month, so numbered that the first two digits of the number show the year of issue, the last two the position in the yearly series. (For example, No. 2201 is the first bulletin of the year 1922.) These comprise the official publications of the University, publications on humanistic and scientific subjects, bulletins prepared by the Division of Extension, by the Bureau of Economic Geology, and other bulletins of general educational interest. With the exception of special numbers, any bulletin will be sent to a citizen of Texas free on request. All communications about University publications should be addressed to University Publications, University of Texas, Austin.

UNIVERSITY OF TEXAS PRESS, AUSTIN

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PUBLISHED BY THE UNIVERSITY FOUR TIMES A MONTH, AND ENTERED A SECOND-CLASS MATTER AT THE POSTOFFICE AT AUSTIN, TEXAS, UNDER THE ACT OF AUGUST 24, 1912

The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar

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This Bulletin is open to the teachers of mathematics in Texas for the expression of their views. The editors assume no responsibility for statements of facts or opinions in articles not written by them.

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FOREWORD

This issue of the Mathematics Teachers' Bulletin is devoted largely to a discussion of the lack of correlation in mathematics that now exists between high school and college. The gap between these two units in our educational system has been widening for a number of years. There are probably many causes of the difficulty which we hope to study in these pages, and, if possible, offer some suggestions that will tend towards discovering a remedy. It cannot be that college teachers are too severe in their requirements, because this is not true of all of them, nor can it be that high-school teaching is poor. The complaint is too general to be caused by an evil so easily explained as by poor teaching. Good teaching will overcome many difficulties, but good teaching may be spoiled by a poor curriculum or by a poor arrangement or sequence of subjects in that curriculum. Whatever the cause may be, the evil has grown up in the past fifteen or twenty years and its growth has been gradual and not a sudden break. Whenever the college student finds his weakness due to poor instruction in the high school, the word generally returns to the home town and the evil is corrected. But when it is due to some trend or drift in educational aims or ideals, he is face to face with something more difficult to explain.

We wish to present in this issue of the bulletin different viewpoints of the "missing link" now found between high school and college, and if possible locate the difficulty. But it will be noticed that this issue of the bulletin is written largely from the viewpoint of the college teacher and may not present the whole of the difficulty. The teachers on the other side of the "gap" should be allowed to have a word in the discussion, and hence we beg our good friends of the high schools to give us in a later issue of the bulletin their viewpoint, making whatever suggestions they may think proper for the good of mathematics in both high school and college.

There seems to be a general demand, at present, for a revision of the high-school course of study, and some discussion has already been made in regard to the revision of the mathematical part of the curriculum. In order to bring out the various opinions of those interested, the pages of the next issue of our bulletin will be devoted largely to a discussion of a new course of study for mathematics in the high school. Our friends and teachers of mathematics in the high schools are urged to send their suggestions and contributions to the editor, whose address is attached to this foreword. Any suggestions made in regard to the relation between high school and junior college will also be gladly received.

C. D. RICE,
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ALGEBRA

About three years ago Andrew Carnegie gave $108,000 for the purpose of investigating the status of engineering education. W. E. Wickenden was appointed director of investigations and in June, 1925, at Schenectady, New York, his committee made the following report:

"The numbers of students attending our secondary schools have increased correspondingly with attendant ill effects upon the scholastic standards of those schools due to overcrowding and too limited staffs. More of these same poorly prepared graduates of high schools than formerly find it possible to attend college and thus the problem of overcrowding and consequent lack of fundamental preparation has been pushed up into the very gates of our institutions of higher education. For privately endowed colleges and universities the solution of this difficulty may be readily found, but for state-supported institutions the way out is not so clear.

"Another element which adds much to the complexity of the problem of admissions is that of the new emphasis laid upon vocational and physical culture subjects in the high schools. The result is that the character of the pre-engineering school training has been greatly modified. It appears that candidates who will offer themselves in years to come will be less, rather than better, prepared to undertake rigorous college mathematical and scientific work.

"The large number of elective subjects in the high school curriculum is confusing to the average boy. Many subjects of his selection do little to develop intellectual power and fitness for college work. They absorb abnormal amounts of the student's time, much to the detriment of his fundamental courses. The defense of this system presumably is that the high school's business is to prepare for life's work and not for college work."

On account of the lack of preparation in algebra, many of the best engineering schools in the country are introducing non-credit courses in algebra at the very beginning of the freshman year. At the University of Wisconsin and Ohio State University a preliminary course in algebra is given consisting of about ten recitations or lectures with numerous problems. At the end of the review in algebra an examination is given on the subject and those who fail to pass this examination are required to take a course in ordinary algebra without credit.

A few years ago the writer wrote to three teachers of mathematics at the A. and M. College of Texas and three at the University of Texas, all six of whom were teaching engineering students. The question was "What is the cause of failure in the freshman class in mathematics?" Each one of the six said "lack of preparation in algebra." There are two reasons for this. One is that the algebra is "finished" in the high school at least one year before graduation. If high-school students are coming to the University they should take algebra their very last year and be fresh on it when they reach the University. Another cause is found in the fact that many teachers of algebra do not insist on blackboard work or written work in the solution of the problems. "Slap them against the blackboard" is a motto that could well be followed. After one-third of a century at the University of Texas, and after experience with thousands of young men in engineering, the writer has come to the deliberate conclusion that high-school graduates in Texas were better prepared for engineering in 1900 than they are in 1925.

The teaching of algebra is the foundation of all material progress. There is not a bridge design in the world that is not designed on the principles of algebra. There is not a dam constructed, an irrigation canal built that does not involve the principles of algebra. Every gun designed for the World War, every battleship that carried our soldiers to the foreign field, every flying machine that sailed the sky was built and constructed and designed by the aid of algebraic formulas. Every highway in the State of Texas was laid out and constructed and manufactured by the principles of algebra. Every mill, motor, turbine engine, or pump used in irrigation was designed by the laws of algebra.

THE TRANSITION FROM HIGH SCHOOL TO COLLEGE

In making the transition from high school to college the pupil finds himself, probably, at a greater disadvantage in mathematics than any other subject. A study of the curriculum of the average high school will explain much of the difficulty of the freshman. Arithmetic is dropped in the first year and seldom noticed again. Algebra, as a rule, is completed in the second year and not studied again until the student enters college. Plane geometry is then taken up and given the best place in the high-school curriculum. It is then dropped in the fourth year to give place to solid geometry or to trigonometry and in some cases to both.

Upon entering coollege the student in his freshman work is expected to know and use intelligently the principles of high-school algebra through quadratics. Having had no work in that subject for two years he finds himself at a great disadvantage. After a lapse of more than two years' time he has forgotten much, and what he remembers was learned at an age of more or less immaturity. A majority of high-school graduates upon entering college do not know how to manipulate algebraic signs and symbols. They find fractions with literal denominators too abstract and often cannot detect the factors of the simplest expressions. The methods of simultaneous solution of equations are forgotten and the meaning and use of substitution is unknown. In a certain class of twenty-six this past session, in our freshman work, there were only three who had ever heard of a quadratic, notwithstanding the fact that the course of study sent out by the State Department of Education specifies that the quadratics must be taught in the high schools of our State.

The mistake made in our American high school is that we seem to be unable to blend or unify the different branches of mathematics, and teach each branch in what is termed closed compartments. This makes it necessary to lose all contact with other subjects while one is being taught.

Algebra is given with no reference to geometry and geometry is studied with a very slight use of algebra. When one is studied the other is dropped, to be forgotten. In the first year of college work, algebra is the basic part of everything that is done, yet in most high schools it is not noticed the last two years.

These serious breaks between high schools and college will continue as long as the high school is regarded as a unit within itself with no relation to the college. The recent development in many of our cities of adding the junior college to the high school seems to indicate a feeling on the part of our educators that the present unit is incomplete and that we are drifting into the more natural arrangement of curriculum and method found in the western countries of Europe. If this development becomes general, let us hope that we will be enabled to eliminate the serious and unnatural breaks that now occur between high school and college, and that a more rational development and arrangement of mathematical subjects may be worked out.

The colleges of the country, for the past few years, have made little or no change in the work required of freshmen. But during that time many of the high schools have let the claims of other subjects and interests take the time that was formerly given to mathematics. This decrease of time given to mathematics, together with the poor arrangement of subjects, is producing a gap between high school and college that is growing from year to year, and, hence, the graduates of the high school upon entering the freshman year find themselves at a decided disadvantage, mathematically, to meet the work required in that year. Since there seems to be no inclination on the part of the college to lower its requirement for the first year, there is left the choice between placing preparatory classes in our colleges and universities, or of requiring entrance examinations. Preparatory classes in our higher institutions of learning have never produced a wholesome atmosphere and are generally opposed, and, hence, if the high-school graduate is not to meet the college requirement, the entrance examination seems to be the best solution of the difficulty. This is now being done in some of our northern institutions, especially, for students who wish to take up engineering or other professional courses dependent largely upon mathematics.

In the face of the difficulties in entrance that now confront the college, the teacher of mathematics in the high school is urged to carry out as faithfully as possible the course of study outlined for his school and by all means to give a thorough review of algebra to the pupil during the last year of his school life. This preparation should be given especially to students who wish to be trained in engineering, business administration, or those lines of work in which mathematics is the basic part of all that is done.

LEARNING TO TALK

Recently the funny (?) papers printed the following joke:

Visitor: How is Willie comin' on in school?

Proud Mother: Swell. Willie, say something to the lady in alegbra.

Of course the "proud mother" is supposed to be laughed at for supposing that algebra is a language. But that is just what algebra is. Willie might have said $x+y=10$, and had the visitor been a teacher of mathematics (and a thoughtful one) she would have regarded the exhibition as creditable to both parent and son. The matter would have been no more of a joke than if she had said "say something to the lady in Latin" and the boy had replied "Labor omnia vincit."

Arithmetic has a language all its own. It consists mainly of the arabic system of notation and the signs of operation. We do not usually think of arithmetic as a linguistic study but, in a certain sense, that is what it is. Algebra is an extension of that subject: Instead of being confined to the arabic numerals, algebra uses in addition a wealth of symbols to represent numbers and has some additional symbols of operation. Learning algebra is mainly acquiring facility in translating from the English language into that of algebraic symbols and of the inverse process.

If we know that man earned a certain sum of money in January and another sum in February we would say in ordinary language: *"The sum earned in January added to the sum earned in February will give the amount of his earnings for the two months."* This is rather a mouthful and requires a good deal of time and labor to set down. Suppose that instead of this method of expression we use shorthand and write for "the sum earned in January," the letters S.J., and for "the sum earned in February", the letters S.F., and for "the sum earned in the two months, "S.T." We may then write, using the symbols of arithmetic for addition and equality, S.J.+S.F.=S.T. This can be farther shortened by merely using J, F, and T for the three numbers. We then have J+F=T. Keeping in mind the meaning given the various letters and symbols, this expression tells us all that the long English statement told us and is easier to speak and to write.

The language of algebra is a condensed language. Suppose we know that a father is four times as old as his son. If we decide to use S to represent the son's age and F the father's age we may write F=4S or S=¼F. F+S will stand for the sum of their ages and F—S for the difference. Since F=4S, 4S may be used instead of F and then 4S+S or 5S will be the sum and 4S—S or 3S the difference. If then we know the sum of the ages to be 50 we have 5S=50, S=10, 4S=40, and the age of each is known. The simple language treatment here suggested will entirely remove the bugaboo of setting up the equation necessary to solve a given algebra problem.

Example: A person has two purses containing $72. One purse contains three times as much as the other. How much in each? If we let S be the amount in the small purse then 3S is the amount in the big purse, and 4S is the sum in both together. It is then quite obvious that translating the statement of the problem into our shorthand we may write 4S=72, S=18, 3S=54, and the problem is solved.

By getting well grounded in the "language" aspect of the subject most of the difficulties of problem solving will never arise. Many extensions and applications of this idea will readily occur to any alert and clever teacher.

TO PROSPECTIVE SECONDARY SCHOOL TEACHERS

From a bulletin sent out to prospective secondary school teachers by Brown University we quote the following good advice:

"It is of fundamental importance that prospective teachers of mathematics in secondary schools should make in college an extensive study of the principles underlying secondary school mathematics, in order that they may acquire a firm grasp of the significance of all operations and a clear understanding of the foundations of the subject. Such a knowledge, together with adequate facility in presenting it, presupposes that prospective teachers have pursued their studies considerably beyond the particular subject matter they expect to teach. Only with such preparation can teachers of mathematics answer the questions of able pupils and lay proper foundations for knowledge of a science so fundamental.

"The courses ordinarily recommended for prospective teachers may be grouped under the following heads: algebra, geometry, elementary analysis, analytic geometry and calculus, and the history of mathematics.

"In the course in college algebra such topics as permutations and combinations, probability, complex numbers, and theory of equations are taken up. The course entitled Fundamental Problems in Algebra includes an introduction to infinite processes, and to some of the modern concepts of number, in the light of which a reconsideration of fundamental operations is undertaken.

"The course in solid and spherical geometry cultivates the ability to reason accurately from fundamental axioms and to solve 'originals.' In the study of foundations, methods, and problems of geometry the student is taught by systematic methods of attack on problems, and introduced to rigorous discussion of the more delicate and difficult parts of the subject, such as systems of axioms. He is also made acquainted with the elementary notions of non-euclidean and four-dimensional geometries and with some simple aspects of relativity. Geometric possibilities are further developed in a semester course on projective geometry treated synthetically.

"The four semester courses in elementary analysis consist mainly of an introduction to the concepts and methods of analytic geometry and calculus. Effort is constantly made to show the unity of various branches of mathematics and to indicate its manifold applications in other sciences and in everyday life.

"The two semester courses in analytic geometry and calculus constitute a natural continuation of those in elementary analysis. In them the student acquires facility in handling the mathematical methods to which he has already been introduced. About half the year is devoted to an intensive study of plane analytic geometry and to a brief treatment of analytic geometry of three dimensions. The rest of the year is devoted to refining and extending the introductory knowledge of calculus gained by the student in the earlier courses.

"In the semester course on the history of elementary mathematics the student learns what contributions different countries have made to the development of mathematics from the earliest times to the present.

"It is highly desirable that prospective teachers should take as much work in mathematics as possible. Among the courses which will make an appeal to the student's interests and needs are theory and construction of geometrical models, descriptive geometry, spherical trigonometry, mathematical theory of statistics, and mathematical theory of investment. In selecting such courses the student is advised to consult some member of the department. It is further recommended that college courses in education, physics, or in physics and chemistry, be taken."

THE GERMAN GYMNASIUM

The gymnasium is the typical secondary school in Germany. The curriculum extends about two years beyond that of the American high school, and in the amount of work done, it corresponds very closely to a continuation of our high school and the first two years of the standard American college. The course of study of these schools has been worked out with great care and is an evolution, so to speak, of several centuries. It is a product of the social life and ideals of the people which it serves and for that reason it would be impossible to transplant it into the life of another people. But there are many points of interest in the aims and methods of the gymnasium that are helpful and instructive to educators in other countries who are working out their ideals and methods of education among their own people.

It was my privilege in the autumn of 1924 to visit one of the best of these schools in Germany. The building and equipment was very much like that of our best high schools, but the faculty and methods used reminded me more of an American college. The faculty was made up entirely of men who had undergone years of rigorous training to fit them for the work they were doing. The idea with them is that a man can teach well only when he has been well taught. This applies to method as well as subject matter.

The arrangement and sequence of subjects in the course of study for the gymnasium is planned to accomplish the greatest amount of work for the time involved. Professor Young, of the University of Chicago, found that, before the World War, the gymnasium could accomplish its work in mathematics in four-sevenths of the time it took the American system to do the same work. Each subject is introduced and carried along progressively and no subject when once begun is allowed to lapse or be forgotten with a loss of time and energy in review when taken up again. There are no breaks as in our high schools, where algebra is dropped and forgotten while geometry is being studied.

In mathematics the methods used and the development of the subject matter were very interesting. The teacher develops first in class the principles under consideration and later the text is used by the student more to assist in rewriting his notes and to correct certain points not well understood in class. In many cases there are two texts used, one developing the principles and the other containing problems only. In no case is the text followed slavishly by pupil or teacher. The first draft of the notes taken by the pupils is a marvel of neatness compared to some notebooks found in our high schools.

The lesson is more often introduced by a skillfully directed set of questions given by the teacher in such a manner that the principle is developed before the class. Generally one member of the class is called to the board and asked to answer on the board the successive questions, the others at their seats carefully taking notes. The whole class works in this way in unison and each one at his seat is ready at any time to take the place of the one at the board and continue the lesson without a break. The lesson is often begun by a set of problems not in the text but carefully devised by the teacher in order to lead to the principle in question and to illuminate the problems of the text. With this carefully planned introductory part of the lesson, one is struck with the amount of work that is accomplished in one hour. It is here one finds the value of the carefully trained teacher and his *complete mastery* of the subject matter.

This high skill and preparation on the part of the teacher leads one, naturally, to inquire into the preparation of that teacher. It is found that the prospective teacher, after finishing the course at a gymnasium, must attend a university three years. He must then undergo a series of rigorous oral and written examinations. This determines his scholarship. *This must come first* and then afterwards the training for his profession. After the work of the university he is to attend a practice school for one year, usually at some good gymnasium where a few are taken each year to be trained by the faculty in charge. During this year he is required (1) to study the practice and theory of teaching, (2) observe the teaching of others, (3) teach himself under the observation of an expert. He must at the end of this year undergo more severe tests. He is then given a trial year at little or no salary to prove his fitness for the high position as a teacher. Here his personality, peculiarities, mannerisms, etc., are thoroughly scrutinized. If he is finally accepted he is practically sure of a life position with a pension ample for old age.

There is given here, in what follows, one of the many outlines of mathematics in the gymnasium. It is not reproduced here with the expectation that it will be followed as a whole in any of our high schools, but with the hope that it may be suggestive to those who are seeking a more rational and logical sequence of topics and subjects in the development of our high-school mathematics and especially useful do we hope it to be to the junior college that is to be the continuation of one or more good high schools to which it is closely affiliated.

MATHEMATICAL CURRICULUM FOR GYMNASIA

A. LOWER STAGE

SEXTA. VI.

The fundamental rules with whole numbers, concrete and abstract, within a narrow range. German measures, weights and coinage. Exercises in the decimal system and in the simplest decimal calculation as a preparation for fractions.

QUINTA. V.

Calculation. Continued exercises in calculation with concrete decimal numbers with an extension of the range of the measures employed (also foreign weights and coinage), measures of length of different kinds (out of doors); simplest problems of surface, and volume-calculation laying stress on the connection between volume and weight. (In such calculations a rough estimate of the magnitude of the result is always to be made.) Numerical factors. Vulgar fractions (first as concrete numbers).

Preliminary geometry. Introduction to the fundamental ideas of space as it may be observed, in such a way however that space appears chiefly as involving plane properties. Dimensions, surfaces lines, points, explained first in relation to immediate objects and illustrated from widely different bodies. Plane figures first as part of the boundaries of bodies, then as independent forms in which the ideas of direction, angle, parallelism, symmetry are to be brought out. Exercise in the use of ruler and compass, constant drawing and measurement.

QUARTA. IV

Calculation. Decimal fractions, contracted methods (in simplest examples). Rule of Three avoiding all exaggeration of stereotyped forms. Problems from commercial life especially simple cases of percentage (interest, discount). Preparation for algebra teaching by repetition of suitable problems previously solved, using letters in place of definite numbers. Meaning of formulae and their evaluation by substitution of special values. Coonnection between rules for mental arithmetic with the rules for brackets.

Geometry. Properties of straight lines, angles and triangles; variation of figures in shape and size; dependence of the parts of a triangle one upon the other; transitional cases (right angled triangles, isoceles triangles, equilateral). Simplest properties of parallelogram deduced from the construction of figures.

UNTERTERTIA. III B

Arithmetic and algebra. Systematic coördination of the fundamental rules of arithmetic by formulae. Conception of relativity developed in practical examples and illustrated by series of natural numbers indefinitely produced in both directions. Computation rules for relative magnitudes. Continuation of exercises in evaluation of formulae, introducing negative quantities and continually insisting on the functional character of the resulting variations in the quantities. Application to pure and complex equations of the first degree with one unknown. Difference between identities and equations.

Geometry. Extension of the parallelogram properties. The trapezium. Fundamental properties of the circle. Consideration of the effect produced on the whole figure by alterations in the length and breadth of various parts. Constructions closely connected with the curriculum omitting all problems soluble only by artifice.

OBERTERTIA. III. A.

Arithmetic and algebra. Completion and extension of algebra, particularly the expansion of polynomials. Simplest proposition in proportion. Pure and complex equations of the first degree with one or more unknowns. Dependence of an expression upon one of its own variables. Graphic representation of simple linear functions and the use of this method in the solution of equations.

Geometry. Comparison of areas and their calculation introducing figures with more complex rectilinear boundaries. Approximate calculation of curvilinear areas. Revision of geometrical calculation dealt with in quinta. Problems as in untertertia.

UNTERSEKUNDA

Arithmetic and algebra. Powers and roots. Pure and complex equations of the second degree with one unknown. Connections between coefficient and roots. Consideration of the variation of a quadratic expression in one variable due to the changes in that variable with graphical representation. Solution of second degree problems with one unknown by intersection of straight line and parabola. Consideration of graphical representation as a means of visualizing relations discovered empirically.

Geometry. Similarity, with special stress on similar situation. Proportion in the circle. Calculation of approximate values for circumference and area of the circle by approaching limiting values of inscribed and circumscribed polygons. Exhaustive investigation of the mutual relation between length of side and size of angle in the triangle, verification of tables for this relation (as preparation for trigonometry). Practical problems arising therefrom (data with plane table).

B. Oberstafe

Obersecunda. II. a

Arithmetic and algebra. Extension of the index idea; conception of the power as exponential magnitude; notion and use of logarithms. Arithmetical progression (of the first order) and geometrical progression; use of the latter in compound interest and annuities (in the simplest examples from actual life). Graphic representation of the mutual dependence of a number and its logarithm. Slide rule; solution of quadratic equations with two unknowns, both by calculation and by graphic representation.

Geometry trigonometry. Deduced from constructive geometry. Use in practical problems of measurement of triangles and quadrilaterals. Concrete expressions of the mutual relation between changes of angle and changes of trigonometrical function by formulae of trigonometry; graphical representation of this treatment of suitable problems in several different ways, by construction and with the help of calculation. Investigation of harmonic relations and the principles of modern geometry as the conclusion of plane geometry.

Unterprima

Arithmetic, etc. Résumé of functions dealt with, having regard to their rise and fall over their whole course (with the possible introduction of the ideas of differential and integral calculus), making use of numerous examples from geometry and physics, especially mechanics. Simplest propositions of the theory of combinations with some exercises.

Geometry. Solid geometry with reference to the most important elements of projection. Exercises in drawing of solids. Simplest propositions of spherical trigonometry. Mathematical geography, including theory of map projection.

OBERPRIMA

(1) Conic-sections treated both analytically and synthetically with applications to the elements of astronomy.

(2) Review work taken from the whole province of school mathematics and where possible, harder problems which have to be solved by calculation and drawing.

(3) Retrospective work introducing historical and philosophical points of view.